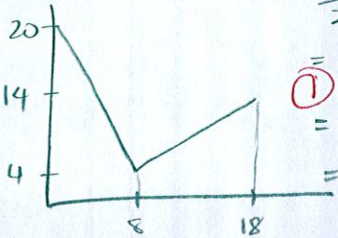
A person's velocity (in meters per minute) at time t (in minutes) is given by $v(t) = \begin{cases} 20 - 2t, & 0 \le t \le 8 \\ t - 4, & 8 \le t \le 18 \end{cases}$. **SCORE: _____ / 5 PTS**

[a] Find the exact distance the person travelled from time $t = 0$ seconds to $t = 18$ seconds.
NOTE: You must show the arithmetic expression that you used to get your answer.



$\frac{1}{2}(20+4)(8-0) + \frac{1}{2}(4+14)(18-8)$

$= \frac{1}{2}(20+4)(8) + \frac{1}{2}(4+14)(10)$ 1 1

$= 96 + 90$

$= 186\text{m}$ $\frac{1}{2}$

[b] Estimate the distance the person travelled from time $t = 0$ seconds to $t = 18$ seconds using three subintervals and right endpoints.
NOTE: You must show the arithmetic expression that you used to get your answer.

$\Delta t = \frac{18-0}{3} = 6$

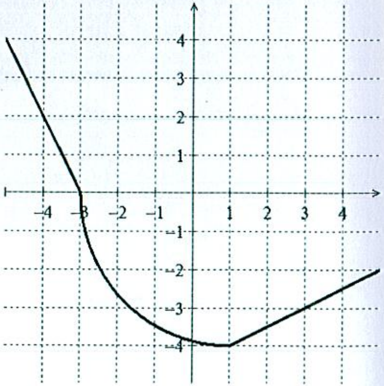
$v(6)\Delta t + v(12)\Delta t + v(18)\Delta t$

$= (8 + 8 + 14)(6)$ 2

$= 180\text{m}$ $\frac{1}{2}$

The graph of function f is shown on the right. **SCORE: _____ / 4 PTS**

<u>The graph consists of a diagonal line, an arc of a circle, then another diagonal line.</u>



[a] Evaluate $\displaystyle\int_{-5}^{5} f(x)\,dx$.

<u>NOTE: You must show the arithmetic expression that you used to get your answer.</u>

$\frac{1}{2}$ $\underbrace{\frac{1}{2}(2)(4)}, \underbrace{-\frac{1}{4}\pi(4)^2}_{1}, \underbrace{-\frac{1}{2}(4+2)(4)}_{1}$

$= \underbrace{-8-4\pi}$

$\frac{1}{2}$

[b] Evaluate $\displaystyle\int_{1}^{-5} f(x)\,dx$.

$= -\displaystyle\int_{-5}^{1} f(x)\,dx = -\left[\frac{1}{2}(2)(4) - \frac{1}{4}\pi(4)^2\right] = \underbrace{4\pi - 4}$ 1

NO POINTS FOR $4-4\pi$

Using the limit definition of the definite integral, and right endpoints, find $\int_{-3}^{-1} (3x^2 + 15x + 18)\, dx$.

SCORE: _____ / 10 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\Delta x = \frac{-1 - -3}{n} = \frac{2}{n}$$

$$1 \quad \underbrace{\lim_{n\to\infty} \sum_{i=1}^{n}}\, f\left(-3 + \frac{2i}{n}\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \underbrace{\frac{2}{n}}_{1} \sum_{i=1}^{n} \underbrace{\left[3\left(-3 + \frac{2i}{n}\right)^2 + 15\left(-3 + \frac{2i}{n}\right) + 18\right]}\ 2$$

$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(-\frac{36i}{n} + \frac{12i^2}{n^2} + \frac{30i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \underbrace{\left(-\frac{6i}{n} + \frac{12i^2}{n^2}\right)}\ 2$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(-\frac{6}{n} \sum_{i=1}^{n} i + \frac{12}{n^2} \sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty} \frac{2}{\cancel{n}} \left(-\frac{\cancel{6}^{\,3}}{n} \underbrace{\frac{\cancel{n}(n+1)}{\cancel{2}}}_{1} + \frac{\cancel{12}^{\,2}}{n^2} \underbrace{\frac{n(n+1)(2n+1)}{\cancel{6}}}_{1}\right)$$

$$= 2(-3+4) \quad 1$$

$$= \underbrace{2}_{1}$$

1 FOR HAVING $\lim_{n\to\infty}$ ON EACH LINE THAT STILL INVOLVES "n"

Evaluate $\int_{-4}^{4} (|x-3| - 7\sqrt{16-x^2})\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE: _____ / 5 PTS**

<u>NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.</u>

$$= \underbrace{\int_{-4}^{4} |x-3|\,dx - 7\int_{-4}^{4} \sqrt{16-x^2}\,dx}_{2} = \underbrace{\tfrac{1}{2}(7)(7) + \tfrac{1}{2}(1)(1)}_{1} \underbrace{-7\left(\tfrac{1}{2}\pi(4)^2\right)}_{1}$$

$$= \underbrace{25 - 56\pi}_{1}$$